

## **ESTABLISHING THE STIFFNESS COEFFICIENT OF A STRUCTURE BY APPROXIMATING THE DECELERATION CURVE WITH POLYNOMIAL LAWS OF VARIOUS DEGREES**

**Soica Adrian, Ispas Nicolae**  
Transilvania University of Brasov, Romania

**Key words:** automobile stiffness structure, approximation, polynomial law

**Abstract** New research paths have been enhanced to cope with passive safety and collision analysis between participants in road traffic and motor vehicle. Engineers attempts to find solutions in order to decrease the injury seriousness suffered by vehicle occupants in accidents. The insurance companies undertake, beside taking over risk and damages evaluation, the risk and evaluation limitation as a main branch of their activity. The hereby paper advances the energetic method used to determine the stiffness coefficient of the touring cars' frontal structure undertaking a frontal collision against a rigid wall. The elements submitted to deformation as a result to the impact are series-connected, therefore the stiffness coefficient of the frontal structure is not constant, varying according to polynomial laws of various degrees. In order to determine the stiffness coefficient the digitization of experimental impact diagrams (acceleration-speed-deformation) is set out.

### **GENERAL OVERVIEW**

Designers typically engineer structures using elastic analysis to withstand service loads without yielding or collapsing. Automotive structures, however, must meet the aforementioned service load requirement; in addition, they must deform plastically in a short period of time (milliseconds) to absorb the crash energy in a controllable manner. They must be light and enable economically mass-production. The automotive safety engineer is responsible for packaging the occupants, so that decelerations transmitted to the occupants may be manageable by the interior restraints to fall within the range of human tolerance. The ultimate goal of the safety engineer is to reduce occupant harm. [2]

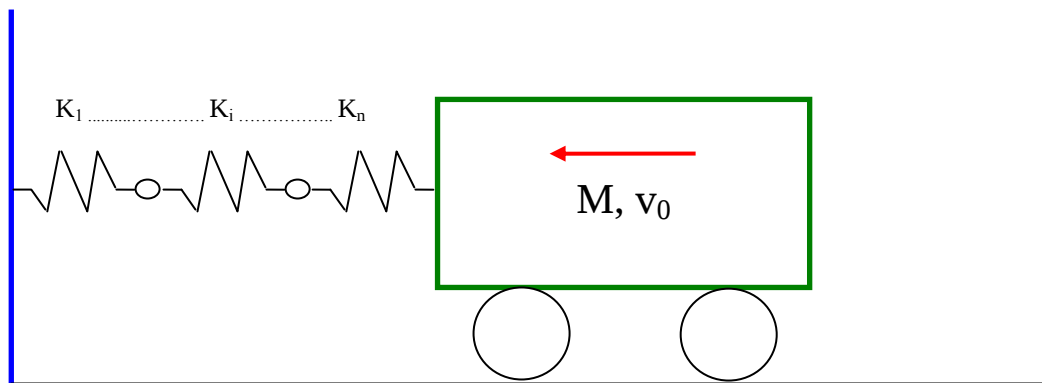
### **SIMULATION CONDITIONS**

To verify the model we initially ana1200 kg, which was subjected to a collision with a deformable barrier at a speed of 15.9 m / s.

The values obtained have been digitized on this curve. The simplified model of the vehicle front-end strength structure is made up of a body mass 'M', moving at speed „ $v_0$ ”, figure 1. The particularity is that elements subject to deformation after impact are connected in series as deformation has different amplitudes and, therefore, the frontal structure stiffness coefficient is not constant, it varies according to certain laws.

Thus, during impact, until the time „ $t_1$ ” it is solely the front elements of the vehicle's body, whose stiffness is „ $k_1$ ”, that deform; during the interval „ $t_1 - t_2$ ” it is the structure elements with „ $k_2$ ”stiffness that deform. After „ $t_{i-1}$ ” until the end of impact „ $t_i$ ” it is the elements whose stiffness is „ $k_i$ ” that deform.

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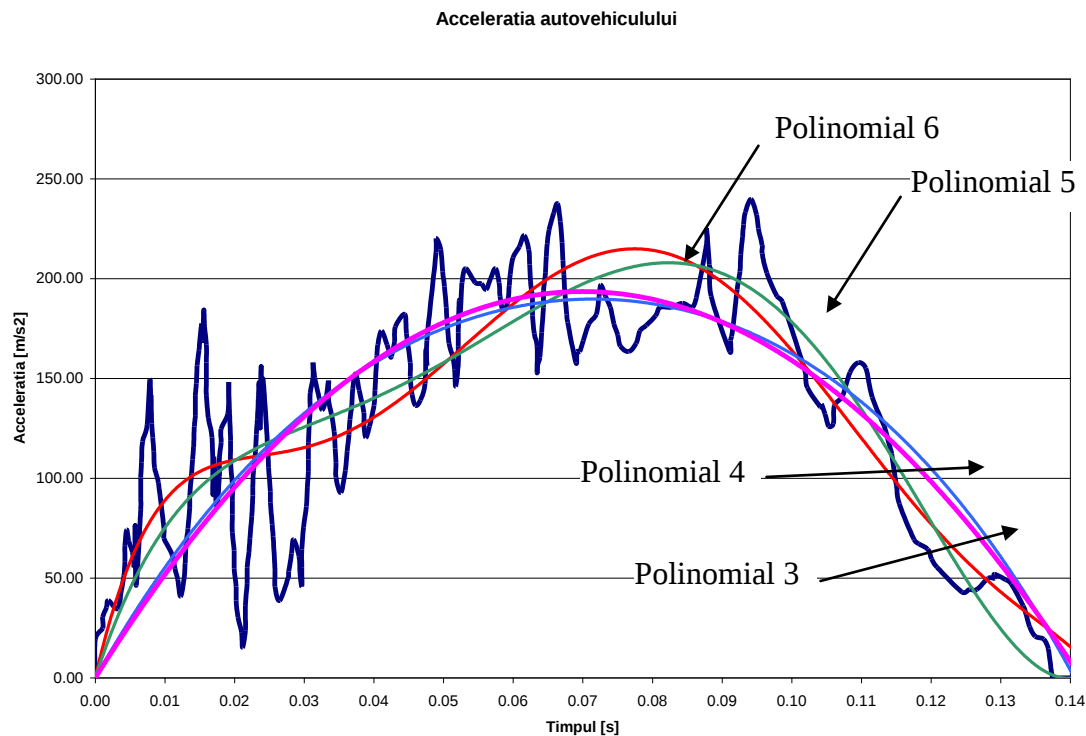


**Figure 1. Simplified model of vehicle front-end structure**

To verify the model we initially analyzed some data recordings in case of a collision, figure 2.

### DECELERATION CURVES APROXIMATION BY POLYNOMIAL LAWS

The analysis of road accidents is often based only on the acceleration diagram of the vehicle structure during the collision. Having the acceleration diagram as starting point, special digitization software applications enable us to electronically obtain the diagram corresponding points. These data also enable us to determine the analytical equations of interpolation polynomials of various degrees. In this paper the acceleration curve was approximated by polynomials of degree 3, 4, 5 and 6. Using two successive integrations of polynomials that describe the acceleration law of variation, the speed and the vehicle deformation according to time is obtained.



**Figure Error! No text of specified style in document.1. Real data and aproximating curves by polynomial laws**

$$a_n(t) = b_0 \cdot t^n + b_1 \cdot t^{n-1} + \dots + b_n$$

$$v_n(t) = v_0 - \left| \int_0^t a_n(t) \cdot dt \right| \quad (1)$$

$$S_n(t) = \int_0^t v_n(t) \cdot dt$$

Where:  $a_n(t)$  – structure deceleration curve;  $b_0 \dots b_n$  – polynomial coefficients describing the acceleration law of variation;  $t$  – impact time;  $v_0$  – initial speed;  $v_n(t)$  – speed variation curve;  $S_n(t)$  – structure deformation curve.

Both absolute values of the results and curves aspect will be taken into account as verifying elements with a view to respecting times at which the speed reaches the value zero and the deformation reaches the maximum value.

### **DETERMINING THE STIFFNESS COEFFICIENT THROUGH ENERGETIC METHODS**

The deformation variation during collision was divided into “ $n$ ” periods, where  $i=1..n$ . Having, by integration, the data regarding the variation of the vehicle speed and the vehicle deformation during impact, we can determine the kinetic energy of the vehicle on the “ $n$ ” iterations.

$$E_{tot} = E_c + E_p \quad (2)$$

At the initial time, before the impact, the vehicle runs at speed „ $v_0$ ”

$$E_{tot} = E_{c_0} = \frac{M \cdot v_0^2}{2} \quad (3)$$

During impact the kinetic energy turns into deformation energy

$$E_p = W_{def} = \frac{k \cdot X^2}{2} \quad (4)$$

From relations (3), (4) and (5) at a given time, during the impact, we can write:

$$\frac{M \cdot v_0^2}{2} = E_c + \frac{k \cdot X^2}{2} \quad (5)$$

$$\frac{k_i \cdot X_i^2}{2} = \frac{M \cdot v_0^2}{2} - E_{c_i} \quad (6)$$

$$k_i = \frac{2 \cdot \left( \frac{M \cdot v_0^2}{2} - E_{c_i} \right)}{X_i^2} = \frac{2 \cdot \left( \frac{M \cdot v_0^2}{2} - \frac{M \cdot v_i^2}{2} \right)}{X_i^2} \quad (7)$$

For each of the iterations „ $i=1..n$ ” the value of the stiffness coefficient „ $k_i$ ” was determined with relation (7). Relation (4) describes the potential deformation energy of the vehicle.

## RESULTS OBTAINED AND CONCLUSIONS

Approximating the deceleration curve by polynomials of various degrees shows that higher degree polynomials render the values recorded accurately in real time.

As the approximating polynomial degree of acceleration increases, both speed and deformation curves are closer to the actual data. Velocity and deformation curves obtained by integration of acceleration data give errors less than 10% as compared to real data.

Briefly, for an approximating polynomial of degree 6 of vehicle deceleration, the steps followed to calculate stiffness are presented in Table 1.

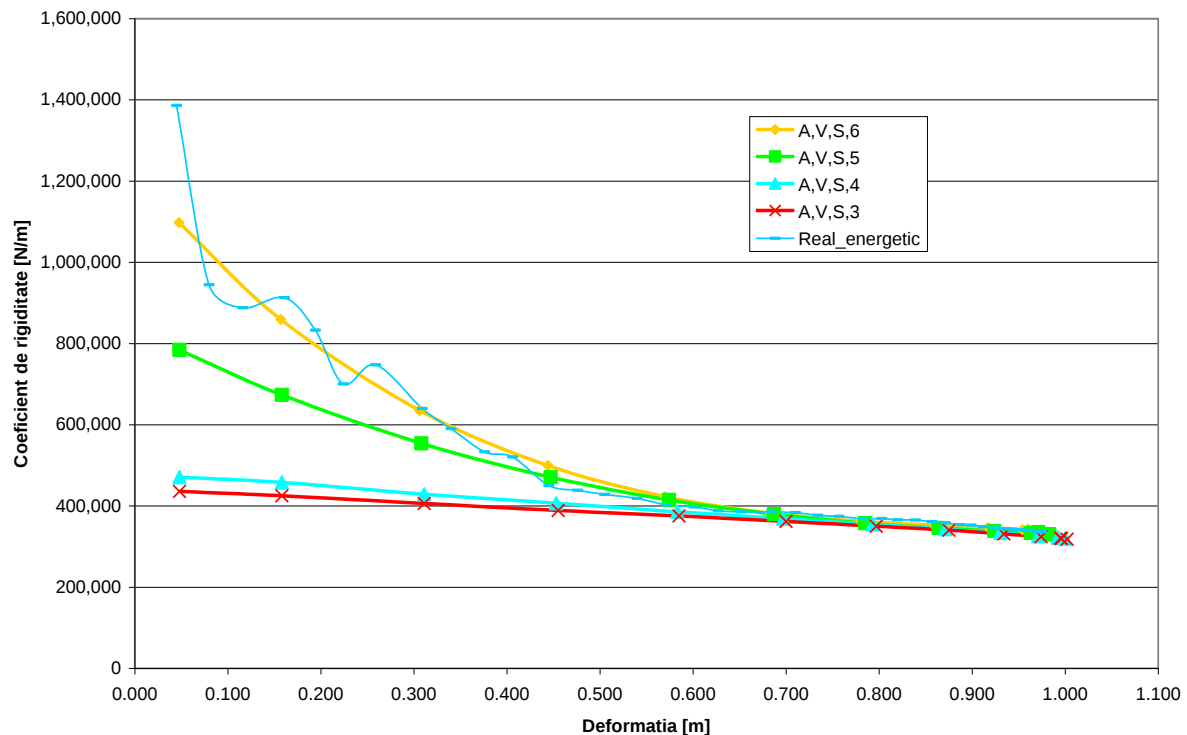
Graphical representation of the stiffness coefficient of vehicle structure taking into account its deformation for various approximating polynomial degrees of the deceleration curve is shown in Figure 3.

The structure stiffness is best approximated by higher degree polynomial laws. Once the initial deformation phase is exceeded, we notice that the polynomial law of degree 6 follows the stiffness curve obtained by processing real data.

**Table 1**

| $i$ | $t_i$ | $X_i$ | $v_i$  | $Ec_i$      | $Ep_i$      | $\Delta Ep_i$ | $\Delta X_i$ | $K$       |
|-----|-------|-------|--------|-------------|-------------|---------------|--------------|-----------|
| -   | [s]   | [m]   | [m/s]  | [J]         | [J]         | [J]           | [m]          | [N/m]     |
| 0   | 0.000 | 0.000 | 15.900 | 159,775.920 | 0.000       |               |              |           |
| 1   | 0.003 | 0.048 | 15.838 | 158,532.298 | 1,243.622   | 1,243.622     | 0.048        | 1,097,752 |
| 2   | 0.010 | 0.157 | 15.364 | 149,185.177 | 10,590.743  | 9,347.121     | 0.109        | 859,324   |
| 3   | 0.020 | 0.306 | 14.345 | 130,052.344 | 29,723.576  | 19,132.834    | 0.149        | 634,875   |
| 4   | 0.030 | 0.444 | 13.224 | 110,520.479 | 49,255.441  | 19,531.865    | 0.138        | 499,710   |
| 5   | 0.040 | 0.570 | 12.005 | 91,083.856  | 68,692.064  | 19,436.623    | 0.126        | 422,851   |
| 6   | 0.050 | 0.683 | 10.573 | 70,650.224  | 89,125.696  | 20,433.632    | 0.113        | 382,113   |
| 7   | 0.060 | 0.780 | 8.848  | 49,477.450  | 110,298.470 | 21,172.774    | 0.097        | 362,585   |
| 8   | 0.070 | 0.859 | 6.854  | 29,689.664  | 130,086.256 | 19,787.786    | 0.079        | 352,594   |
| 9   | 0.080 | 0.917 | 4.720  | 14,079.949  | 145,695.971 | 15,609.715    | 0.058        | 346,528   |
| 10  | 0.090 | 0.954 | 2.640  | 4,404.787   | 155,371.133 | 9,675.162     | 0.037        | 341,431   |
| 11  | 0.100 | 0.970 | 0.817  | 421.853     | 159,354.067 | 3,982.934     | 0.016        | 338,727   |
| 12  | 0.110 | 0.971 | -0.608 | 233.628     | 159,542.292 | 188.225       | 0.001        | 338,429   |
| 13  | 0.120 | 0.960 | -1.589 | 1,595.750   | 158,180.170 | -1,362.122    | -0.011       | 343,273   |

**Abordare energetica**



**Figure 3. Values of stiffness coefficients given the data for the vehicle deceleration curve**

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